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Vortex nucleation in superfluid ^{4}He

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Abstract. W F Vinen suggested in 1961 that the creation of quantized vortex line in superfluid ^{4}He is impeded by an energy barrier. Measurements of the rate ν at which negative ions nucleate vortices in isotopically pure ^{4}He have vindicated this idea. Barrier heights derived from the measured temperature dependence of ν are in good agreement with values calculated by Muirhead, Vinen and Donnelly (MVD) on the assumption that the nascent vortex first appears as a small loop at the equator of the moving ion. The MVD model can in addition provide a satisfactory explanation for the extraordinary sensitivity of ν to tiny traces of ^{3}He observed in experiments. A prediction that vortices can also be created by a fast adiabatic expansion of liquid ^{4}He through the lambda (superfluid) transition—perhaps modelling the creation of cosmic strings at a cosmological phase transition in the early universe—has been tested. The results imply line densities that are smaller by a factor of at least 100 than those predicted. It is pointed out that Vinen's contributions to the understanding of superfluidity have been substantial and that their influence is likely to be felt far into the future.

1. Introduction

It is both an honour and a particular pleasure to have been invited to speak at the meeting celebrating the scientific career and achievements of Joe (W F) Vinen. His name has been familiar to me since my undergraduate days at Queen's University, Belfast, where I was required to read, understand, and answer an examination paper on Atkins' (then new) monograph [1] on liquid helium. It was full of enthusiastic and detailed discussions of the Hall and Vinen experiments on rotating helium [2], which I found absolutely fascinating. Little did I guess that I would later meet, and have the pleasure of getting to know, both Henry Hall and Joe Vinen.

Unlike the other speakers at the meeting, I have never collaborated formally with Joe, or co-authored a paper with him, and to that extent I am an imposter. Nonetheless, as I shall seek to show, we have shared many interests in common over the years, and many of our activities have been complementary and mutually reinforcing. In particular, activities in the Lancaster experimental programme have owed much to Joe's encouragement and insight. Many of our observations were predicted by him, sometimes decades in advance. And he has usually been quick to propose imaginative explanations, I think invariably correct, of unexpected results emerging from the cryostat.

I now propose to address the problem of how vortices are created *ab initio* in superfluid helium. In section 2 I will review very briefly the idea of superfluidity and quantized vortices. I will outline the nucleation problem in section 3, and will explain how it was attacked experimentally in section 4. Possible connections between quantized vortices and the early universe are discussed in section 5, where I will summarize the current status of

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recent ‘cosmological experiments’ on the formation of topological defects in phase transitions. Finally, in section 6, I will seek to draw some conclusions.

2. Superfluidity and quantized vortices

As it cools through the lambda (superfluid) transition, at temperature $T_\lambda = 2.17$ K under the saturated vapour pressure, liquid ^{4}He undergoes a marked change of phase to a new state. The properties of the liquid above and below the transition are so strikingly different that the phases are called He I and He II respectively. He II behaves as though it were a mixture of two distinct but completely interpenetrating fluids: a normal-fluid component that has viscosity and entropy; and a superfluid component with neither viscosity nor entropy. The two fluids are physically real to the extent that e.g. their densities can be measured experimentally [3], and their flow velocities can be separately described and measured [1, 4].

The superfluid component of He II, which is characterized by a macroscopic wave function (a complex scalar order parameter), has some very unusual properties. Its inviscid character means that no pressure head is required to maintain flow through very narrow orifices or tubes and, correspondingly, that a moving object does not experience the usual drag force. Even more remarkable is the fact that the superfluid does not rotate—at least not in a conventional manner.

If a bucket of He II is rotated slowly, the superfluid component remains at rest relative to the fixed stars. For faster rotations, Hall and Vinen showed [2] that quantized vortex lines appear in the liquid, parallel to the axis of rotation. For sufficiently high angular velocities, with a large density of lines, the liquid simulates the solid-body rotation that had been observed earlier by Osborne [6] and—provided one does not look too closely—it acquires a conventional parabolic surface. It is interesting to note that the non-rotation at small angular velocities of the container is a *fundamental* property of the liquid, and not just because it is hard for the container walls to get a grip on something as slippery as a superfluid: Hess and Fairbank showed [5] that, when a slowly rotating container of liquid ^{4}He is cooled through the lambda transition, the nascent superfluid is formed in a state of zero angular velocity.

The most remarkable feature of these vortex lines is their quantization. The possibility was first suggested by Onsager [7], and was enthusiastically endorsed by Feynman [9], but it remained no more than an idea until Joe’s celebrated vibrating wire experiment [8]. As shown in figure 1, he demonstrated beyond all doubt that the circulation

$$\kappa = \oint \boldsymbol{v}_s \cdot \mathrm{d}\boldsymbol{l} = n\left(\frac{h}{m_4}\right) \tag{1}$$

where the integral is taken round a closed loop, $\boldsymbol{v}_s$ is the superfluid velocity, m_4 is the ^{4}He atomic mass, and n is an integer. In practice, he found that either $n = 0$ (i.e. no vortex around the wire) or $n = \pm 1$. This quantization can be seen as a natural consequence of London’s suggestion that the superfluid can be described in terms of a macroscopic wave function [10].

Following the pioneering observations of quantized vortex lines [2, 8] it quickly became apparent that they are by no means confined to rotating He II, but that they can appear under a wide range of different circumstances: in fact, whenever the liquid is treated at all roughly. For example, in addition to the orderly arrays of lines in rotating He II, random tangles of vortex line can be created (always with $n = 1$) when critical velocities are exceeded in flow and thermal counterflow experiments [11–13], by ultrasound [14], and by focused second sound [15]. Quantized vortex rings—rather like smoke-rings, with the charge trapped on the vortex core—can be created by moving ions [16–18]. In fact, as demonstrated by Awschalom and Schwarz [19], it is apparently impossible to prepare a macroscopic sample of He II without

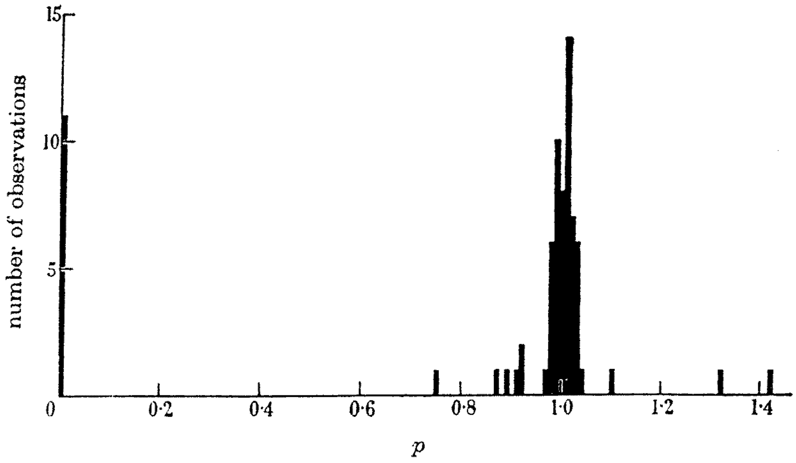


Figure 1. A histogram of measurements by Vinen [8] of the circulation around a vibrating wire immersed in He II. The abscissa unit is the quantum of circulation, $\kappa = h/m_4$.

vortex lines: there seem always to be some *remanent* vortex lines pinned cobweb-like between small protuberances on the walls, in metastable equilibrium. It is notable that, of the hundreds of papers published on the vorticity generated in flow and thermal counterflow experiments, almost all analyse their data in terms of Vinen dimensionless parameters [11].

Donnelly has provided a detailed account [20] of the nature and properties of quantized vortices, including a much fuller set of references than can be given here.

3. The vortex nucleation problem

A great deal is known about the properties of quantized vortices, based on the kinds of experiment mentioned above, as well as many others [20]. We now consider the rather different, but equally important and, in some ways, more fundamental question: where do the vortex lines come from? How do they get into the liquid in the first place? This question cannot be addressed via any kind of bulk experiment, because of the remanent vorticity. The critical velocities observed in flow experiments, for example, refer to the conditions needed for the expansion and growth of pre-existing vortex lines—and not to the creation of vortex lines *ab initio*. One approach to the problem is to study the transition from ions to charged vortex rings: the ions are so small that they are most unlikely to be affected by remanent vorticity.

The experimental programme at Lancaster, for which Joe's ideas and insights have been so valuable, has been based on the so-called negative ion. This is a spherical void in the liquid created by an excess electron. With a radius of $\sim$1 nm and a hydrodynamic effective mass of $\sim 100 m_4$ (both quantities are pressure dependent), the negative ion provides a semi-macroscopic charged probe that can be used to investigate the properties of the liquid. It is easily created by field emission [21, 22], and its charge enables its position and velocity in the liquid to be tracked. It is especially useful as a probe of liquid ^{4}He which, unlike the rare isotope ^{3}He, is magnetically inert.

At this point, a warning is necessary. Many of the early ion experiments yielded inconsistent results, and it was not until 1980 that the reason for this became apparent: the vortex nucleation process is quite extraordinarily sensitive to traces of ^{3}He. Even the $\sim 2 \times 10^{-7}$

parts of ^{3}He in the commercial helium obtained from natural gas are sufficient to change the vortex nucleation rate (see below) by several orders of magnitude [23], as shown by the data in figure 2. Results obtained up to this time therefore refer, in effect, to the properties of extremely dilute ^{3}He–^{4}He solutions of unknown concentration, and not to ^{4}He as had naturally been assumed.

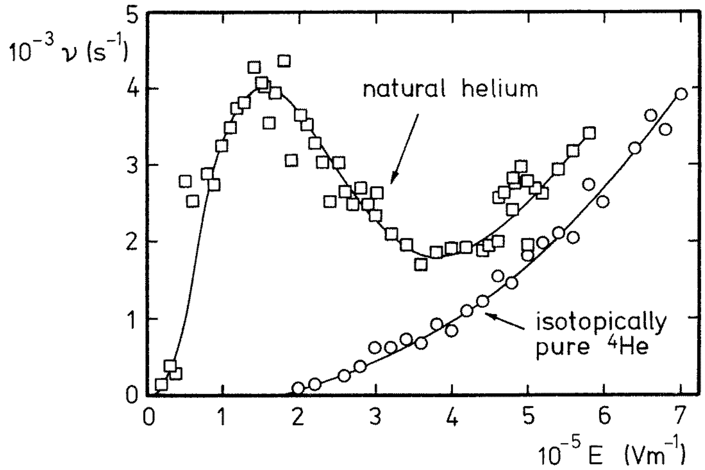


Figure 2. Measurements [23] of the rate ν at which negative ions nucleate vortex rings in He II as a function of electric field E, showing the extraordinary sensitivity of this process to the presence of ^{3}He isotopic impurities: the circle data are for isotopically pure [24, 25] ^{4}He, and the squares are for ordinary commercial helium, which typically contains $\sim 2 \times 10^{-7}$ of ^{3}He. The curves are guides to the eye.

A method was therefore developed [24, 25] for removing the ^{3}He isotopic impurities. It was based on the so-called heat-flush phenomenon, using a 'wind' of normal-fluid component from a heater to blow away the ^{3}He atoms, while collecting the isotopically pure superfluid component. The product ^{4}He is believed to be devoid of ^{3}He. Of course this cannot be proved; but the ^{3}He–^{4}He isotopic ratio has been measured as $<2 \times 10^{-15}$. Remarkably, for such a seemingly useless product, pure ^{4}He produced in this way has also found several other applications. It has been used e.g. to support experiments on 2D ion pools below the superfluid ^{4}He surface [26], 2D electron sheets above the surface [27], quantum evaporation [28], and ultra-cold neutrons [29].

Using isotopically pure superfluid ^{4}He, it became possible to investigate the vortex nucleation process and, in particular, to test Joe Vinen's prophetic suggestion from 1961 [30] that '... the creation of vortex line is opposed by a large potential barrier'. We will consider the corresponding experiments, and how they were accomplished, in section 4.

Perhaps stimulated in part by the Lancaster ion experiments, and in part by a controversy [32, 33] about the way in which the initial vortex appears on the moving ion, Muirhead, Vinen and Donnelly (MVD) calculated Joe's energy barrier [31] for the case of a negative ion. They considered the two possible nucleating geometries sketched in figure 3, with the nascent vortex appearing either as a symmetrically placed encircling ring, or as a tiny loop with its ends pinned to the surface of the ion. In the first case [33] it was supposed that the ion would quickly move sideways and become trapped on the vortex core. In both cases, it was expected that the charged-vortex-ring complex would be stable, and that it would grow rapidly and slow down [17] under the influence of an applied electric field. MVD calculated,

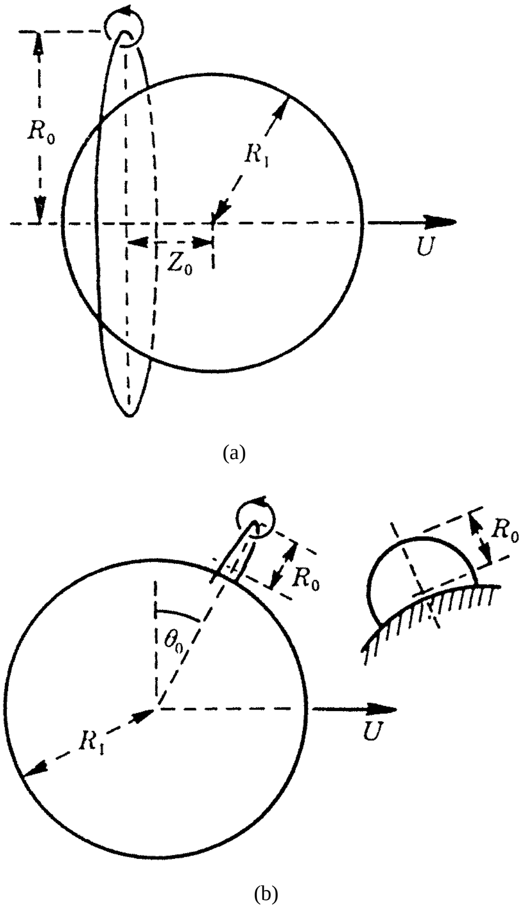


Figure 3. Creation of a charged vortex ring by a moving ion: the two different nucleating geometries considered by Muirhead, Vinen and Donnelly [31]. Either the nascent vortex ring first appears whole, symmetrically placed about the axis of motion (a), or it first appears as a tiny loop whose ends are pinned on the ion (b).

for different initial ion velocities, as a function of the loop/ring radius R, the change in energy ΔE that would occur if the ring/loop were created at constant impulse. Some of their results for loops are shown by the curves in figure 4. At low initial velocities, the process cannot occur while conserving energy. Above a critical velocity of $\sim$60 m s^{-1}, however, nucleation becomes energetically possible. The process is impeded by the effective energy barrier to the left of the point at which the $\Delta E(R)$ curve first crosses (or touches) the $\Delta E = 0$ axis. MVD also performed similar calculations for the other nucleating geometry (creation of an encircling vortex ring) and found very much higher critical velocities, thereby demonstrating the physical implausibility of the latter process.

4. Vortex nucleation experiments

There is a fundamental problem in designing vortex nucleation experiments using ions in a superfluid. The simplest imaginable approach would be somehow to move the ion at a constant

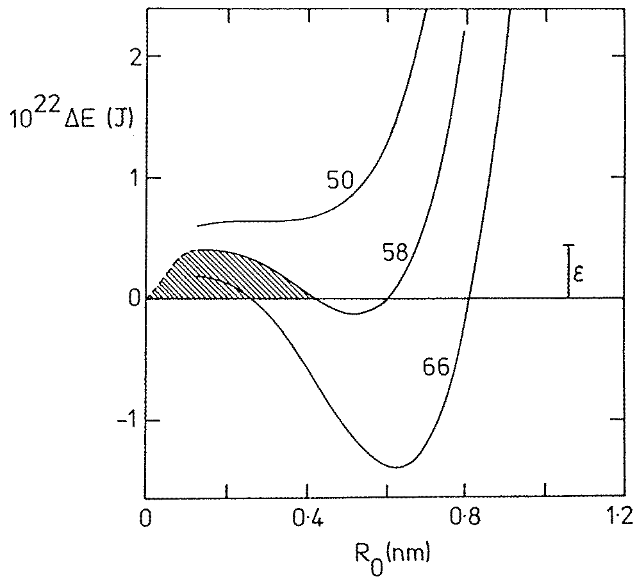


Figure 4. The change in energy ΔE during the creation of a vortex loop at constant impulse, as a function of the loop's radius of curvature R_l, for the different initial velocities shown by the figures adjacent to the curves. Calculated by Muirhead, Vinen and Donnelly [31]. The effective energy barrier for an initial velocity just above the critical value is shaded. For comparison, the measured [42] activation energy for vortex creation is shown by the bar.

speed, to measure the nucleation probability, and then to repeat the experiment for different speeds, pressures and temperatures. But, given that any electric field, however small, will cause the ion to accelerate (almost) without limit, how can one control the ionic velocity? Some of the early experiments [34, 35] achieved control by balancing the force from an applied electric field against the drag force arising from the scattering of excitations (phonons and rotons) at temperatures in the 0.3–1.0 K range. With the clear vision of hindsight, we can now appreciate that this approach was unlikely to lead to useful results because the vortex nucleation rate rises exponentially fast with T (see below) in this range, quite apart from the ^{3}He effect [23] mentioned above.

In the Lancaster experiments, a different approach has been used. The speed of the ion is controlled by balancing the force of the applied field against the rate of momentum loss caused by roton creation above the Landau critical velocity. The technique relies on the assumption, apparently vindicated by the results, that roton creation and vortex nucleation are independent processes. It relies on earlier observations [16, 36] that, provided the He II is under sufficient pressure, negative ions can reach velocities near v_L without immediately nucleating and being trapped on vortex rings, and the subsequent demonstration [37] that even modest electric fields are sufficient to propel the ions at speeds significantly above v_L. Measurements [38] of the dependence of drag on velocity showed that the drag on a moving ion remains negligibly small until v_L is attained, and then rises quickly, as shown in figure 5. (Later, more accurate, measurements [39] showed that, within experimental error, the onset of drag occurs at *precisely* the predicted value of v_L.) The behaviour at pressures below $\sim$11 bar is entirely different. The ions then create charged vortex rings almost immediately because, as it turns out, v_L is then greater than the critical velocity for vortex nucleation. If the ^{4}He is not isotopically purified then, again, the ions create charged vortex rings almost immediately for $T < 0.35$ K, even

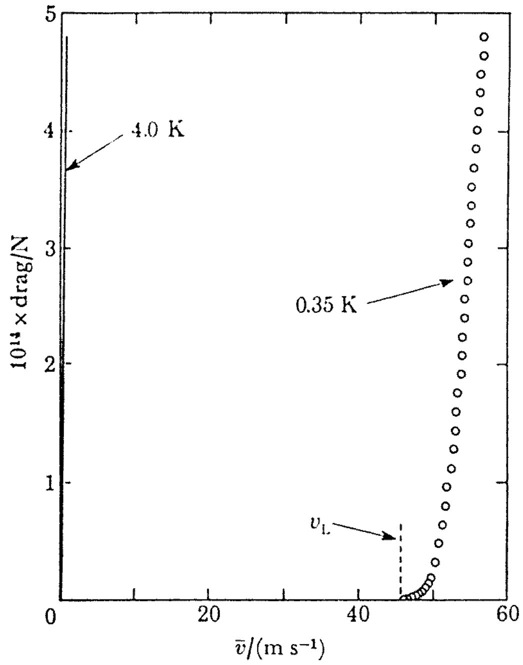


Figure 5. A demonstration [38] of roton emission by ions exceeding the Landau critical velocity v_L in He II. Drag on the moving ion remains negligibly small for velocities below v_L; but, above v_L, it rises rapidly. For comparison, the quite different behaviour seen in He I is shown by the line rising from the origin.

when $P > 11$ bar [40]. But, by using isotopically purified ^{4}He pressurized to $P > 11$ bar, one can readily control the speed of the ions, within the range just above v_L, by adjustment of the applied electric field.

The technique [41] devised for investigating the nucleation process is based on electric induction, using the arrangement sketched in figure 6. As a disk of ions with charge q moves at average velocity $\bar{v}$ between electrodes (approximated as being of infinite area) separated by L, an induction current

$$i_c = \frac{q\bar{v}}{L} \tag{2}$$

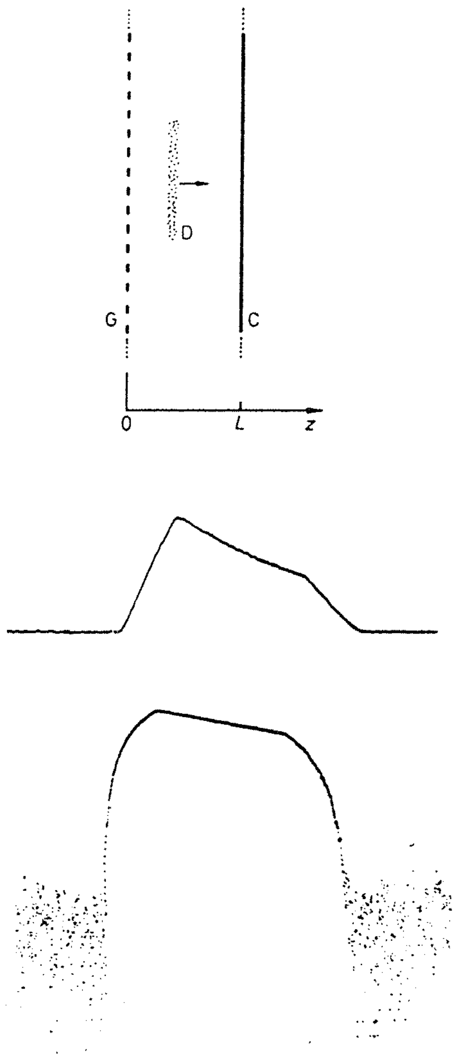
must flow from the collector if it is to be kept at a constant potential. If, however, some of the ions nucleate vortex rings, the charged rings will expand and slow down very quickly indeed. In effect, therefore, the contribution to i_c from any particular ion abruptly disappears when it nucleates a ring. The anticipated probabilistic [34, 35] decay of the ensemble of ions should therefore show up as an exponentially decaying current at the collector

$$i_c = \left(\frac{q\bar{v}}{L}\right)\exp(-\nu t) \tag{3}$$

where ν is the vortex nucleation rate. This is precisely what was observed, as shown in figure 6.

The technique was used to explore a large volume of the multi-dimensional parameter space in terms of T, P, E and the ^{3}He concentration x_3 (down to the 10^{-9} level). A typical set of data [42] for pure ^{4}He is shown in figure 7. It can be seen immediately that ν is constant at

(a)



(b)

(c)

Figure 6. The electric induction technique [41] for measurement of the vortex nucleation rate ν. (a) The electrode geometry (schematic). (b) A typical induction signal; and (c) its logarithm.

low T, but then rises very rapidly with increasing T. The curves represent fits to each set of data of an equation of the form

$$\nu(T) = \nu(0) + A\exp(-\epsilon/k_B T) \tag{4}$$

where $\nu(0)$, A and ϵ are constants. It was found that $\epsilon/k_B = (3.1 \pm 0.8)$ K. This behaviour strongly suggests that the system tunnels through a barrier at low T, but can be thermally activated over it at higher T. The measured barrier height [42] is compared with MVD's prediction [31] in figure 4, where the experimental value is shown by the bar. The agreement is quite remarkably good—indeed even better than could reasonably have been expected, given the approximations inherent in the MVD model. It can be regarded a triumphant confirmation of Joe Vinen's insights [30] of 1961.

The MVD calculations were also extended [43] to encompass the influence of ^{3}He isotopic

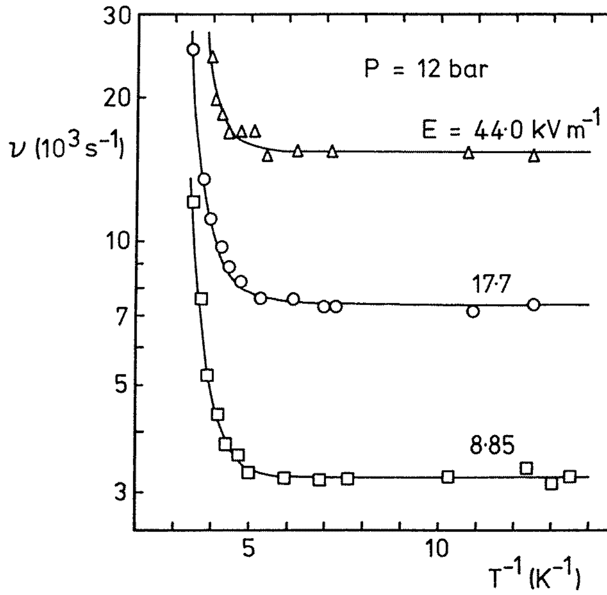


Figure 7. Measurements [42] of the vortex nucleation rate ν as a function of reciprocal temperature T^{-1}, for three electric fields. The sample of isotopically pure ^{4}He was held under a pressure of 12 bar. The curves represent fits of (4) to the data with $\epsilon/k_B = 3.1$ K.

impurities. The underlying idea is that the binding energy of a ^{3}He atom on the outside of the ion [46, 47] might be less than its binding energy on the nascent vortex loop. If so, the atom could liberate energy by transferring from the ion to the vortex, effectively reducing the critical velocity. This approach explained immediately the extraordinary sensitivity of ν to traces of ^{3}He, and it accounted for all the main features of the experiments [44].

5. Quantized vortices and the early universe

It seems at first sight astonishing that there can be any relationship at all between the early universe and the properties of liquid helium. Yet there are some very interesting conceptual connections [48]. Cosmologies based on grand unified theories (GUTs) imply that a symmetry-breaking phase transition took place very early on, at $t \sim 10^{-35}$ s after the big bang, as the universe fell through a critical temperature of $T \sim 10^{27}$ K. In this transition, the Higgs fields acquired finite values, the forces of nature became distinct, and the false vacuum gave way to the true vacuum that we know today. At the same time, it is thought that topological defects in space-time would have created because of the causal disconnection of separated regions, via the Kibble mechanism [49]: e.g. cosmic strings [45] with

Mass $\sim 10^{18}$ tonnes m^{-1}

Radius $\sim 10^{-31}$ m

Length $\sim$ light years

Speed $\sim c$.

Cosmic strings are of particular interest and potential importance because they may have provided the primordial density inhomogeneities on which the galaxies later condensed.

A major problem in assessing the truth or otherwise of this idea is the non-repeatability of the experiment—which has only been run once, when the observable universe was about the size of a grapefruit, and when there was nobody around to see what happened. Zurek pointed out [50] however that, although experiments at 10^{27} K are far beyond any imaginable extension of our present technology, it may be useful to explore mathematically analogous condensed matter systems. He suggested that the similarities between the lambda and GUT transitions are such that an investigation of the former under non-equilibrium conditions may help to illuminate the remote and inaccessible events that may have occurred during the latter. The analogy arises because both transitions can be considered to be of second order, and because the complex scalar order parameter (macroscopic wave function) Ψ of ^{4}He is similar to commonly considered cosmological order parameters, so one can infer correspondences:

$$\begin{aligned} &\text{Higgs field 1} \longleftrightarrow \text{Re}\,\Psi \\ &\text{Higgs field 2} \longleftrightarrow \text{Im}\,\Psi \\ &\text{False vacuum} \longleftrightarrow \text{He I} \\ &\text{True vacuum} \longleftrightarrow \text{He II} \\ &\text{Cosmic string} \longleftrightarrow \text{Quantized vortex line.} \end{aligned}$$

The basic idea, therefore, was to take liquid ^{4}He through the lambda transition *fast*, and to see whether vortex lines were created in the process. Zurek realized that the large specific heat of liquid ^{4}He near T_λ precludes the possibility of cooling it quickly through the transition, but that there was no reason why it should not be expanded very rapidly through the transition as sketched in figure 8. He also made estimates of the vortex line densities to be expected, as a function of speed through the transition.

Zurek's ideas were realized first by exploiting, not the lambda transition, but (weakly first-order) phase transitions in liquid crystals [51, 52]. The 'bulk version' [53] of the proposed

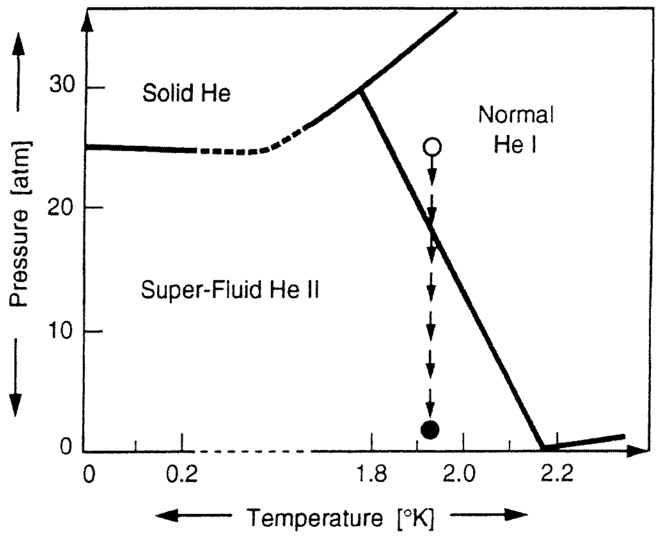


Figure 8. Zurek's suggestion [50, 53] (schematic) for a cosmological experiment in liquid helium. A sample of liquid ^{4}He is expanded rapidly through the lambda transition, and is expected thereby to create vortex lines as an analogue of cosmic strings. (Note that the actual expansion trajectories are not isothermal [55].)

^{4}He experiment was later carried out using a specially designed expansion apparatus [54], and seemed to reveal that large densities of vortex lines are created at the transition, just as predicted. An unexpected observation in the initial experiments [54] was that small densities of vortices were created even for expansions that occurred wholly in the superfluid phase, provided that the starting point was very close to T_λ. The phenomenon was initially [55] attributed to vortices produced in thermal fluctuations within the critical regime, but Joe pointed out [56] that effects of this kind are only to be expected for expansions starting within a few μK of the transition, i.e. much closer than the typical experimental value of a few mK. The most plausible interpretation—that the vortices in question were of conventional hydrodynamic origin, arising from non-idealities in the design of the expansion chamber—was disturbing, because expansions starting above T_λ traverse the same region. Thus some, at least, of the vortices seen in expansions through the transition were probably not attributable to the Kibble–Zurek mechanism as had been assumed. It has been of particular importance, therefore, to undertake a new experiment with as many as possible of the non-idealities in the original design eliminated or minimized.

An ideal experiment would be designed so as to avoid all fluid flow parallel to surfaces during the expansion. This could in principle be accomplished by e.g. the radial expansion of a spherical volume, or the axial expansion of a cylinder with stretchy walls. In either of these cases, the expansion would cause no relative motion of fluid and walls in the direction parallel to the walls and presumably, therefore, no hydrodynamic production of vortices. The walls of the actual expansion chamber [54, 55] were made from bronze bellows, thus approximating the cylinder with stretchy walls. Although there must, of course, be some flow parallel to surfaces because of the convolutions, such effects are relatively small. It is believed that the significant non-idealities, in order of importance, arose from: (a) expansion of liquid from the filling capillary, which was closed by a needle valve 0.5 m away from the cell; (b) expansion from the shorter capillary connecting the cell to a Straty–Adams capacitive pressure gauge; (c) flow past the fixed yoke on which the second-sound transducers were mounted. In addition (d) there were complications caused by the expansion system bouncing against the mechanical stop at room temperature.

The experiment has therefore been repeated [57], taking appropriate measures to minimize these problems. The new expansion cell is sketched in figure 9. The main changes from the original design are as follows: (a) the sample filling capillary is now closed off at the cell

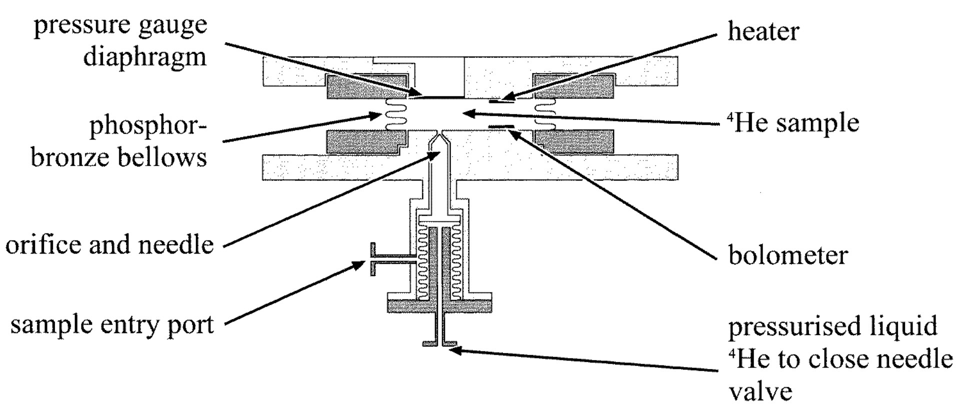


Figure 9. The improved expansion cell [57] used to minimize flow parallel to solid surfaces in the 'bulk version' [53] of Zurek's experiment.

itself, using a hydraulically operated needle-valve; (b) the connecting tube to the pressure gauge has been eliminated by making its flexible diaphragm part of the chamber end-plate; (c) the second-sound transducers are also mounted flush with the end-plates of the cell, eliminating any support structure within the liquid; (d) some damping of the expansion was provided by the addition of a (light motor vehicle) hydraulic shock-absorber.

Following an expansion though the transition, a sequence of second-sound pulses is propagated through the liquid. If the anticipated tangle of vortices is present, the signal may be expected to grow towards its vortex-free value as the tangle decays and the attenuation decreases. Signal amplitudes measured just after two such expansions are shown by the data points of figure 10. It is immediately evident that, unlike the results obtained from the original cell [54], there is now no evidence of any systematic growth of the signals with time or, correspondingly, for the creation of any vortices at the transition. One possible reason for this is that the density of vortices created is smaller than the theoretical estimates [50, 53, 58], but we must also consider the possibility that they are decaying before they can be measured: there is a 'dead period' of about 50 ms after the mechanical shock of the expansion, during which the resultant vibrations cause the signals to be extremely noisy (which is why the error bars are large on early signals in figure 10.

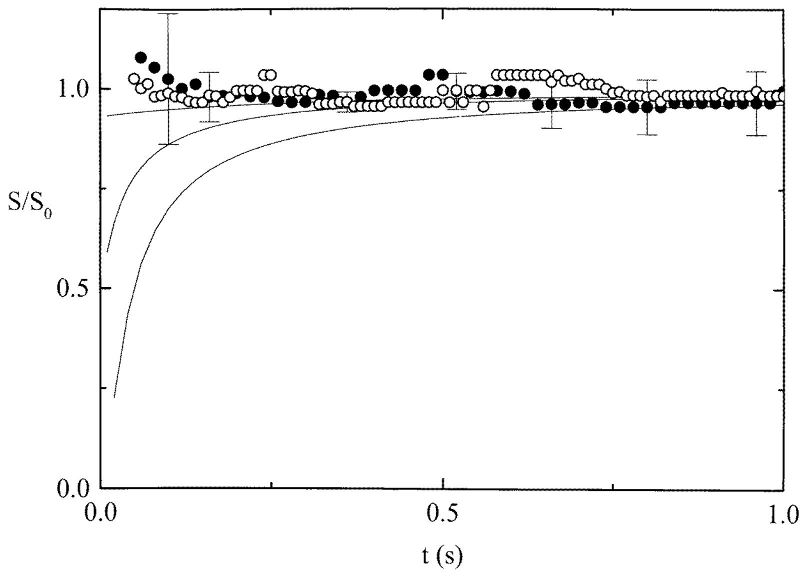


Figure 10. Evolution with time t of the second-sound amplitude S [57], following an expansion of the cell at $t = 0$. The data are scaled by the amplitude S_0 of the second sound in the absence of vortices. They were derived from two different expansions along the same trajectory, starting above the lambda line. The curves show the signal evolutions to be expected for initial line densities of, from the bottom: 10^{12}, 10^{11} and 10^{10} m^{-2}.

The rate at which a tangle of vortices decays in this temperature range is determined by the Vinen [11] equation

$$\frac{\mathrm{d}L}{\mathrm{d}t} = -\chi_2 \frac{\hbar}{m_4} L^2 \tag{5}$$

where L is the length of vortex line per unit volume, m_4 is the ^{4}He atomic mass and χ_2 is a dimensionless parameter. The relationship between vortex line density and second-sound attenuation is known [20] from experiments on rotating helium, and may for present purposes be written in the form

$$L = \frac{6c_2 \ln(S_0/S)}{B\kappa d} \tag{6}$$

where c_2 is the velocity of second sound, S_0 and S are the signal amplitudes without and with vortices present, B is a temperature-dependent parameter, $\kappa = h/m_4$, and d is the transducer separation.

Integrating (5) and inserting (6), one finds immediately that the recovery of the signal should be of the form

$$\left[\ln\left(\frac{S_0}{S}\right)\right]^{-1} = \frac{6c_2}{\kappa B d}\left(\chi_2 \frac{\kappa}{2\pi} t + L_i^{-1}\right). \tag{7}$$

Of the constants in (7), all are known except χ_2 and B which seem not to have been measured accurately within the temperature range of interest. A subsidiary experiment [57], deliberately creating vortices by conventional means and then following their decay by measurements of the recovery of the second-sound signal amplitude, allowed χ_2/B to be determined. This measured value was then used to calculate the evolution of S/S_0 with time for different values of L_i, yielding the curves shown in figure 10. From the quench time $\tau_Q = (17 \pm 1)$ ms measured during the expansion, and Zurek's estimate [58] of

$$L_i = \frac{1.2 \times 10^{12}}{(\tau_Q/100\ \text{ms})^{2/3}} \qquad (\text{m}^{-2}) \tag{8}$$

we are led to expect that $L_i \approx 4 \times 10^{12}$ m^{-2}. A comparison of the calculated curves and measured data in figure 10 shows that this is plainly not the case. In fact, the data suggest that L_i is smaller than the expected value by at least two orders of magnitude.

In the light of the apparently positive outcome of the earlier experiment [54], this null result has come as a considerable surprise. It is worth commenting, first, that Zurek's estimates of L_i were never expected to be accurate to better than one, or perhaps two, orders of magnitude, and his more recent estimate [59] suggests lower defect densities. So it remains possible that his picture [50, 53, 58, 59] is correct in all essential details, and that the improved experiment with faster expansions now being planned will reveal evidence of the Kibble–Zurek mechanism in action in liquid ^{4}He. Secondly, however, it seems surprising that comparable experiments on superfluid ^{3}He [60, 61] seem to give good agreement with Zurek's original estimates [50, 53, 58] whereas the present experiment apparently shows that they overestimate L_i by at least two orders of magnitude. It is not yet known for sure why this should be. Thirdly, an interesting explanation of the apparent discrepancy in terms of thermal fluctuations changing the winding number has recently been suggested [62] by Karra and Rivers.

6. Conclusions

The ion experiments have demonstrated *inter alia* that

(a) vortex nucleation is impeded by an energy barrier, as inferred [30] by Joe in 1961;
(b) nucleation involves tunnelling through the barrier, or thermal activation over it, and the experiments [42] have confirmed the barrier height calculated [31] by Muirhead, Vinen and Donnelly;
(c) the extreme sensitivity of the nucleation rate to traces of ^{3}He arises because the ^{3}He atom is more strongly bound to the nascent vortex loop than to the ion, and the experiments [44] yield results in good agreement with calculations [43] by Muirhead, Vinen and Donnelly based on this idea.

The main conclusion from the experiments on vortex creation at the lambda transition must be that further work is needed, both experimental and theoretical. But it can also be concluded that vortices generated through the Kibble mechanism, if any, appear at densities lower by a factor of at least 100 than the initial predictions [50, 53]. Finally, I would like to quote a couple of lines from Elgar's oratorio *The Apostles* [63] '. . . and our name shall be forgotten in time, and no man have our work in remembrance . . .'.

This must of course be true for all of us. But Elgar says nothing about the time constant characterizing this forgetting! Good science, which is influential and gets into the standard texts and monographs, is characterized by a long time constant. I am entirely confident that Joe Vinen's time constant will prove to be very long indeed.

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First performed in Birmingham, section IV, from the words given to Judas in his remorse and despair after the betrayal.